MATH 1150 **Polynomials!** **Name:**____________________

Precalculus February 12, 2019

Purpose: In this problem set, you will be working towards the goal of graphing complicated polynomials and wacky rational functions. To that end, you will work first with monomials.

Definitions: A **polynomial** is a function that can be written as

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n$$

where $a_0, a_1, \ldots, a_n$ are constants (called **coefficients**). A **rational function** is a quotient of polynomial functions (but some are sneaky—more on this later).

Which if the following functions are polynomials? Which are rational functions?

1. $f(x) = -x^3 + x^2 - 1$
2. $f(x) = x^{\frac{3}{2}} + x^2 - 1$
3. $f(x) = \dfrac{x^3}{x^2 - 1}$
4. $f(x) = \dfrac{ax^2 + bx + c}{ax^2 + bx + c}$
5. $f(x) = (x-3)^3$
6. $f(x) = \dfrac{(x-1)(x-2)(x+3)}{\pi x}$

More Definitions: The definitions below reference the polynomial

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n$$

- The highest power, n, of x in the above polynomial is called the __________ of $f(x)$.
- The number multiplied by this largest power of x, a_n, is called the ____________________.
- The number that is multiplied by x^0 is called the ____________________.

Consider $g(x) = 5x^3 - x + 1$.

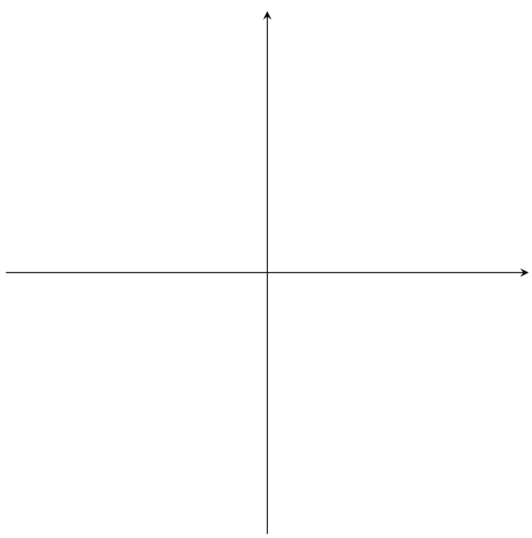
1. What is the degree of g?
2. What is the leading coefficient?
3. What is the constant term?
4. What is the y-intercept?
5. What is the domain of g?

Our goal is to understand polynomials but those seem tough. Let's start with something smaller.

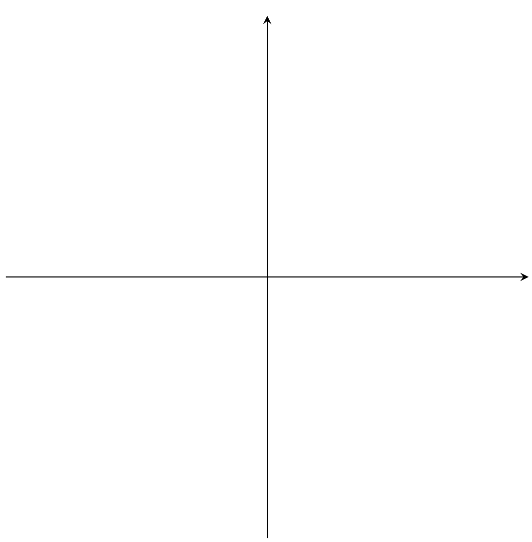
Definition: A **monomial** is a polynomial with a single term, or a function of the form

Monomials fit into two categories: ________________. These match the definition we learned for even and odd functions but it's easier to remember because of the following connection:

- $f(x) = ax^n$ is **even** if n is ____________________.



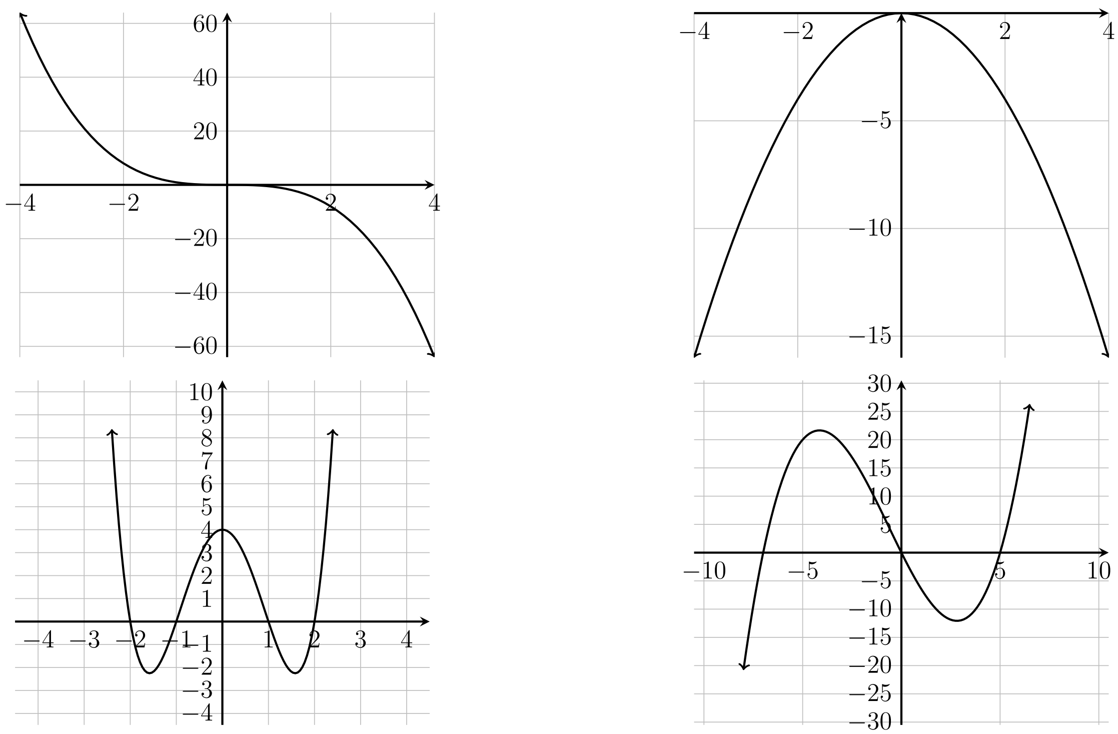
- $f(x) = ax^n$ is **odd** if n is ____________________.



A polynomial is just a finite sum of monomials (see Calculus 2 for infinite sums).

Definition: The **leading term**, $a_n x^n$ of the polynomial is the monomial corresponding to the highest power of x. This monomial will determine the end behavior of the WHOLE polynomial.

For each graph below, is the degree of the polynomial even or odd? Is the leading coefficient positive or negative?



Definition: The horizontal intercepts of polynomials are called **zeros** or **roots**.

Circle the roots of the polynomials graphed above.